

Data-Driven Physics

DFT and ML are vital for testing material properties in pharmaceuticals, clean energy, and engineering industries.

- The Kohn-Sham equations are the basis to Density Functional Theory (DFT):

$$\left[-\frac{1}{2}\nabla^2 + V_n(r)\right] \phi_i(r) = \epsilon_i \phi_i(r)$$

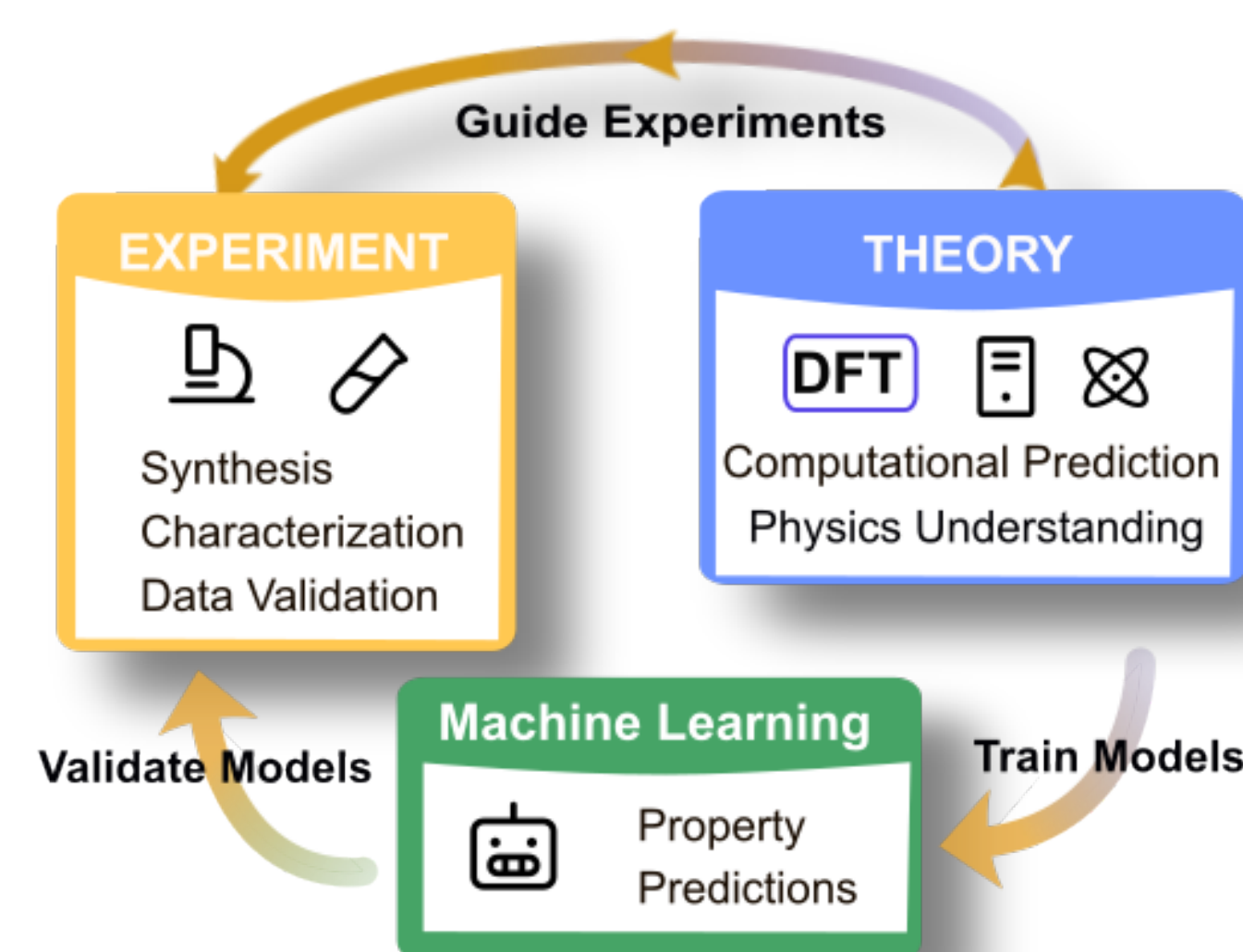
$$n(r) = \sum_i |\phi_i(r)|^2$$

$$V_H(r) = \int \frac{n(r')}{|r - r'|} dr'$$

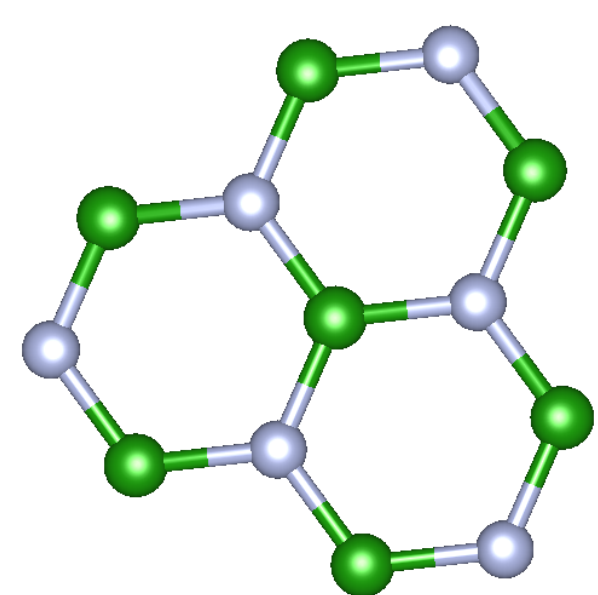
$$\left[-\frac{1}{2}\nabla^2 + V_n(r) + V_H(r)\right] \phi_i^{\text{new}}(r) = \epsilon_i^{\text{new}} \phi_i^{\text{new}}(r)$$

Iterate

- Physics from DFT, accuracy from Machine Learning (ML): Replace costly band gap corrections with learned corrections.



- The modified Becke-Johnson (mBJ) method band gaps closely approximate experimental values ($\sim 0.1\text{--}0.3\text{ eV}$ accuracy) and serve as the prediction target.
- Case Study: Hexagonal Boron Nitride (hBN) — monolayer

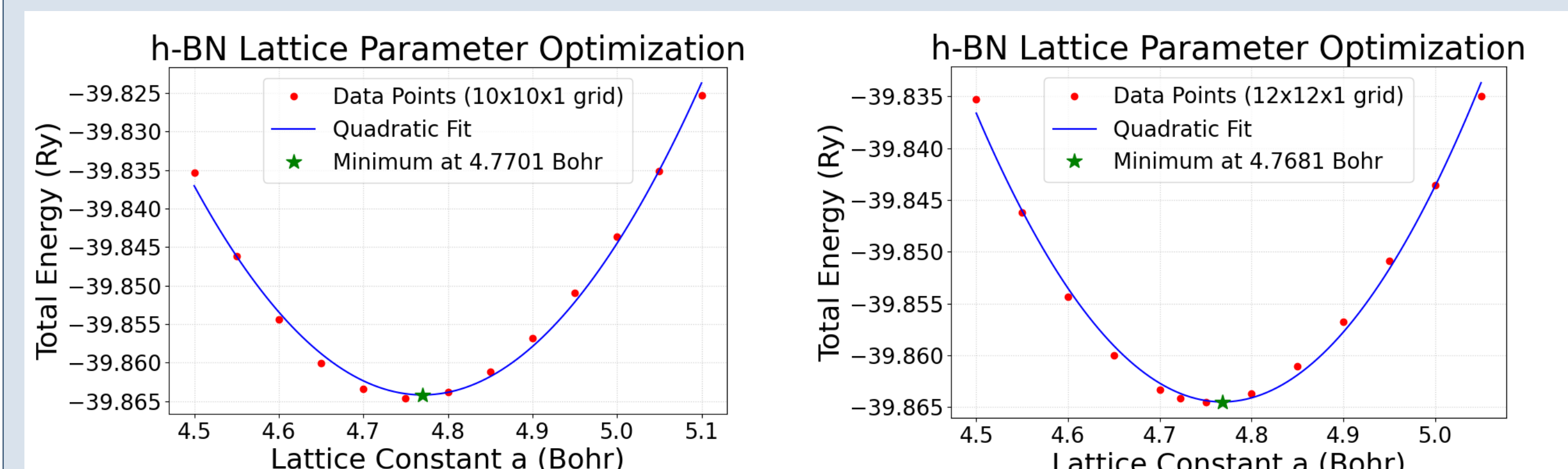


- Honeycomb lattice with alternating boron and nitrogen atoms.
- Ultrawide band gap insulator (about 6 eV).
- Insulating substrate and encapsulation layer for two-dimensional devices.

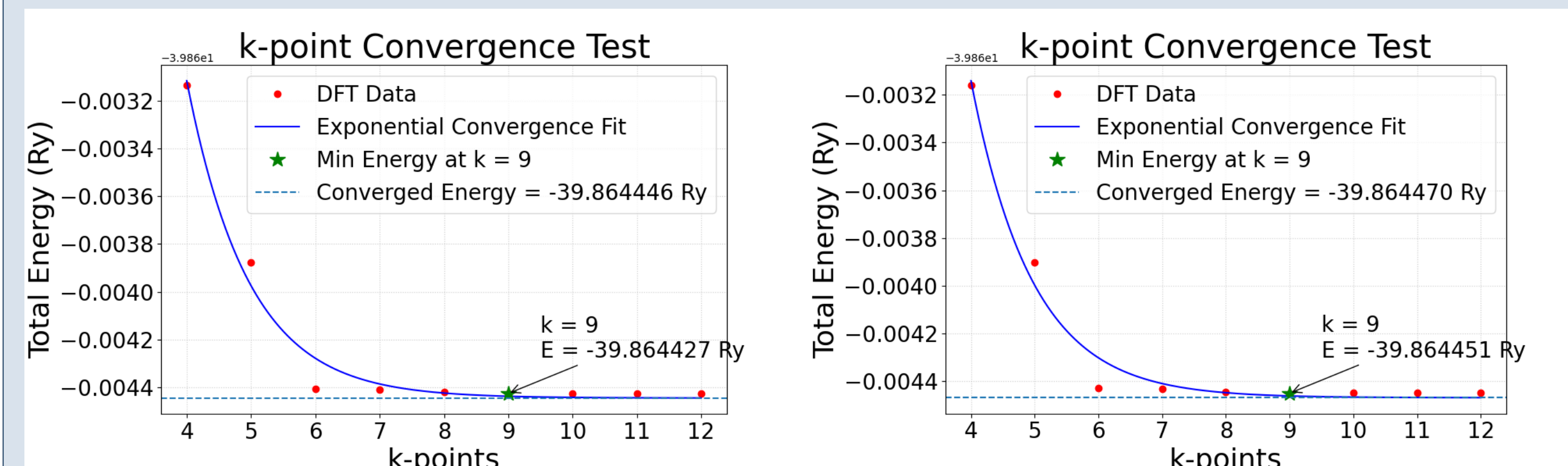
DFT Foundation

The graphs below illustrate the results of the calculations used to extract computational parameters for h-BN with k-point grids of 10x10x1 and 12x12x1 respectively.

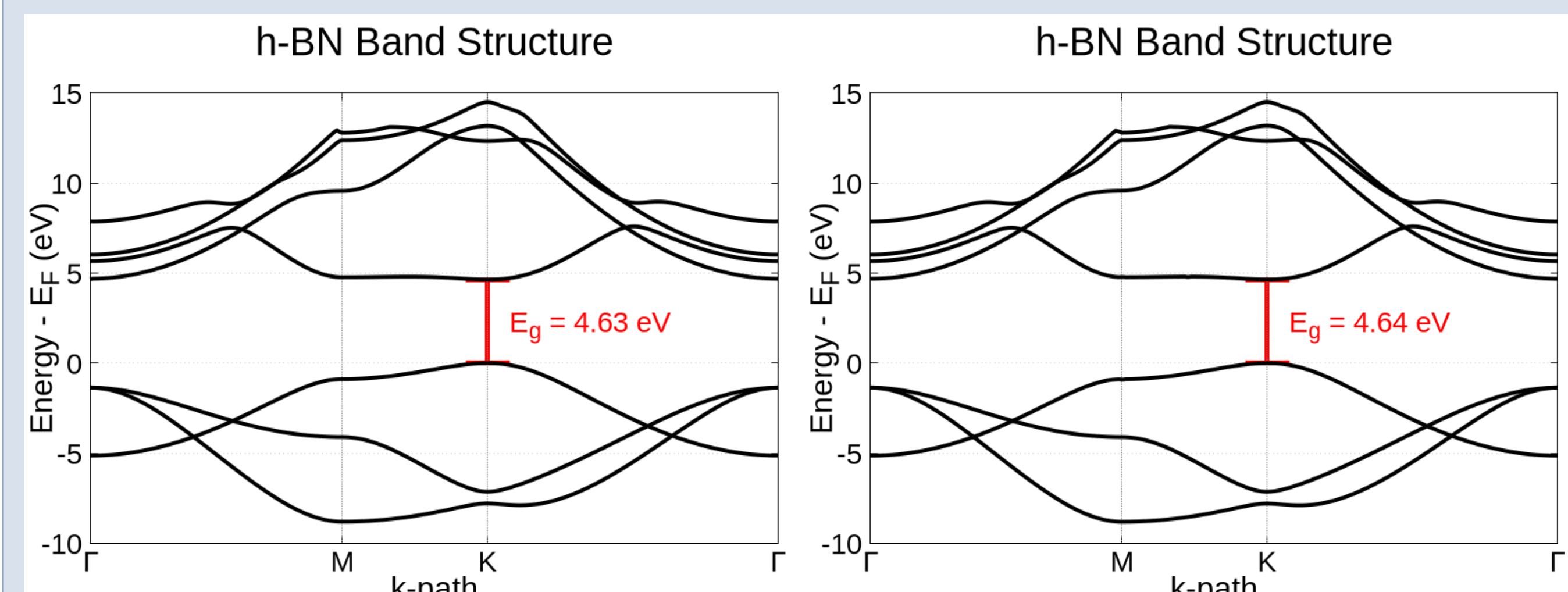
- Lattice parameter optimization from total energy minimization.



- Energy convergence with respect to k-point sampling.



- Standard PAW-DFT Band Structure of Monolayer h-BN



- The optimized lattice constant is $a = 4.7$ Bohr ($\approx 2.49 \text{ \AA}$), in excellent agreement with the accepted value for monolayer hBN ($\approx 2.50 \text{ \AA}$).
- Accepted/experimental-scale value is $\approx 6.08\text{ eV}$, while our standard DFT (PAW) result is 4.6 eV (typical underestimation by baseline DFT).

ML Correction

The workflow for obtaining and comparing MBJ data

Download data $\rightarrow$ 2D Materials (JARVIS)
 ~ 1100 materials $\rightarrow$ mBJ filtering $\rightarrow$ 246 materials

Train/Test Split $\rightarrow$ Training Set (80%) $\rightarrow$ Test Set (20%)
 All non-hBN Hold-out non-hBN + hBN

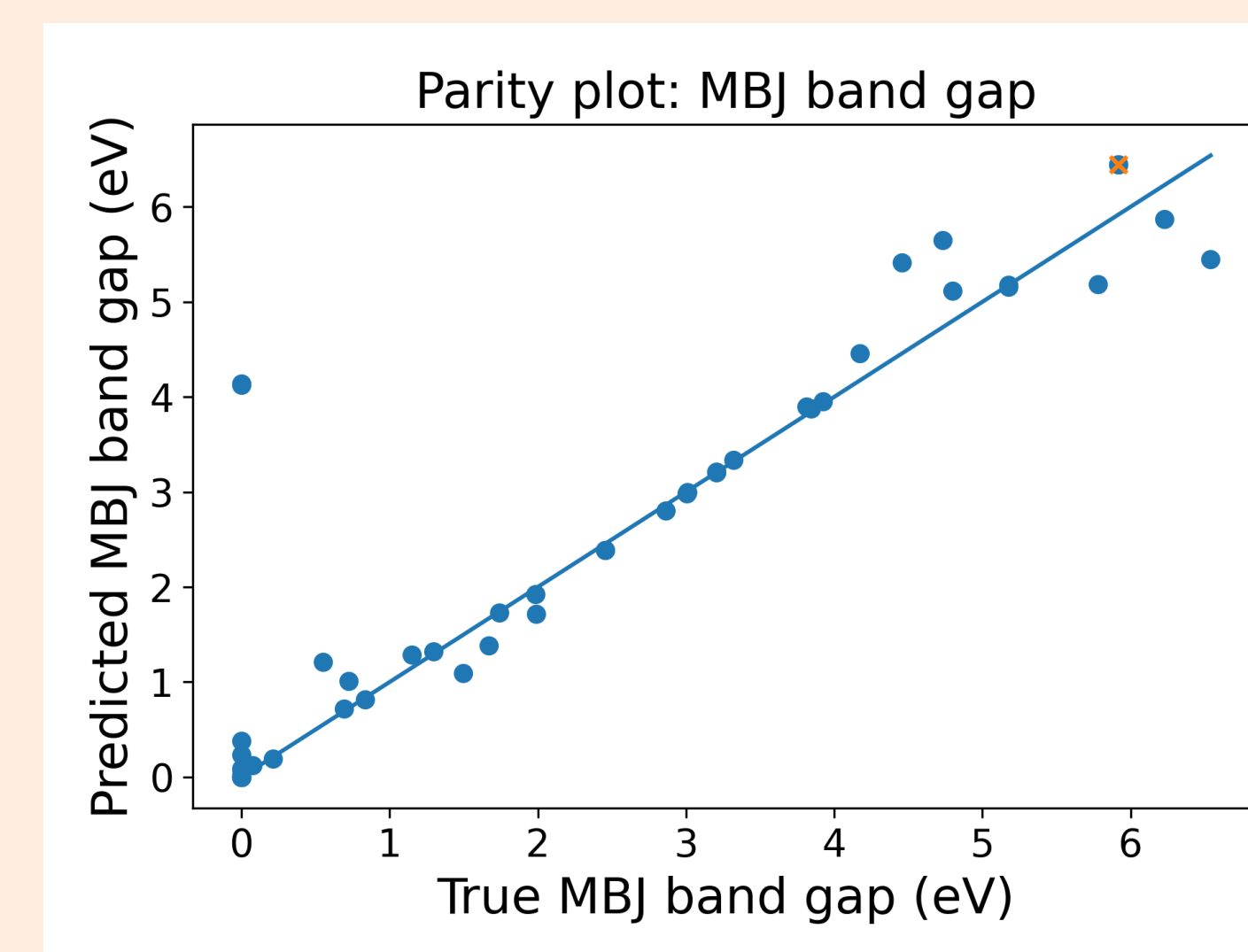
Features (X) $\rightarrow$ Physics features Composition descriptors
 2 features:
 $\diamond$ Standard DFT gap
 $\diamond$ Electronic dielectric
 Prediction: ~ 5.23 eV
 22 chemical features:
 $\diamond Z$: atomic number
 $\diamond X$: electronegativity
 $\diamond R$: atomic radius
 $\diamond IE$: first ionization
 Prediction: ~ 6.0 eV

Prepare Data $\rightarrow$ Convert to numeric
 Mean Imputation
 Build $X_{\text{train}}, Y_{\text{train}}$

Train + Evaluate $\rightarrow$ Random Forest Regressor

Metrics: MAE, R^2

Predict E_g^{MBJ}



- Overall model error (MAE): 0.336 eV

- Overall fit (R^2): 0.938

- Held-out monolayer hBN test (not used in training): The model predicts E_g^{MBJ} close to the reference value.

- JVASP-76649: $E_g^{\text{MBJ}} = 5.917$ eV, Predicted $E_g^{\text{MBJ}} = 6.049$ eV.

Analysis of Results

Density Functional Theory

- The standard DFT band gap is underestimated ($\sim 4.6\text{ eV}$ vs. $\sim 6.08\text{ eV}$), consistent with the expected band-gap error of conventional DFT.
- Energy vs. lattice parameter optimization yields an equilibrium lattice constant of ~ 4.7 Bohr, in very good agreement with the accepted experimental value for monolayer hBN.

Machine Learning

- The model achieved MAE = 0.336 eV and $R^2 = 0.938$ using 246 monolayer materials with MBJ band-gap references.
- Including composition descriptors improved the predicted hBN band gap from 5.23 eV (DFT-only features) to 6.0 eV, much closer to the MBJ value of 5.917 eV.

Overall

- Standard DFT captures the essential physics of monolayer hBN, but systematically underestimates its band gap.
- Machine learning uses DFT-based features and composition descriptors to learn this correction, giving near-mBJ band gap predictions at much lower computational cost.
- Future work will expand the dataset and explore more advanced machine learning models to further improve band gap predictions for 2D materials.

Acknowledgments

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Selected References

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